લિબર્ટી પેપરસેટ

ધોરણ 12 : ગણિત

Full Solution

સમય : 3 કલાક

અસાઈનમેન્ટ પ્રશ્નપત્ર 15

PART A

1. (B) **2.** (B) **3.** (D) **4.** (C) **5.** (D) **6.** (C) **7.** (B) **8.** (C) **9.** (B) **10.** (A) **11.** (B) **12.** (D) **13.** (A) **14.** (D) **15.** (C) **16.** (D) **17.** (C) **18.** (A) **19.** (D) **20.** (A) **21.** (D) **22.** (D) **23.** (A) **24.** (D) **25.** (A) **26.** (C) **27.** (B) **28.** (C) **29.** (B) **30.** (C) **31.** (C) **32.** (B) **33.** (D) **34.** (D) **35.** (C) **36.** (C) **37.** (D) **38.** (C) **39.** (D) **40.** (C) **41.** (C) **42.** (A) **43.** (B) **44.** (B) **45.** (B) **46.** (C) **47.** (B) **48.** (D) **49.** (C) **50.** (B)

PART B

વિભાગ-A

1.

$\tan^{-1}\left(\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right)$

$= \tan^{-1}\left(\sqrt{\tan^2\dfrac{x}{2}}\right)$

$= \tan^{-1}\left(\left|\tan\dfrac{x}{2}\right|\right)$

$= \tan^{-1}\left(\tan\dfrac{x}{2}\right)$

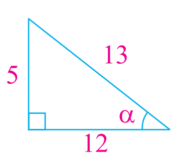
$\left(\because\ 0 < x < \pi \Rightarrow 0 < \dfrac{x}{2} < \dfrac{\pi}{2}\ \therefore\ \tan\dfrac{x}{2} > 0\right)$

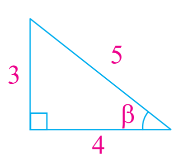
$= \dfrac{x}{2}\ \left(\because\ \dfrac{x}{2} \in \left(0, \dfrac{\pi}{2}\right) \subset \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)\right)$

2.

ડા.બા. $= \cos^{-1}\dfrac{12}{13} + \sin^{-1}\dfrac{3}{5}$

$\cos^{-1}\dfrac{12}{13} = \alpha \quad , \quad \sin^{-1}\dfrac{3}{5} = \beta$





$\therefore\ \cos\alpha = \dfrac{12}{13}, \sin\alpha = \dfrac{5}{12}\ \Big|\ \sin\beta = \dfrac{3}{5}, \cos\beta = \dfrac{4}{5}$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\dfrac{5}{13}\right)\left(\dfrac{4}{5}\right) + \left(\dfrac{12}{13}\right)\left(\dfrac{3}{5}\right)$

$= \dfrac{20}{65} + \dfrac{36}{65}$

$= \dfrac{56}{65}$

$\therefore\ \alpha + \beta = \sin^{-1}\dfrac{56}{65}$

$\therefore\ \cos^{-1}\dfrac{12}{13} + \sin^{-1}\dfrac{3}{5} = \sin^{-1}\dfrac{56}{65}$

3.

$y = \cos x^3 \cdot \sin^2(x^5)$

$\dfrac{dy}{dx} = \cos x^3 \cdot \dfrac{d}{dx}\sin^2(x^5) + \sin^2(x^5)\dfrac{d}{dx}\cos x^3$

$= \cos x^3 \cdot 2\sin(x^5) \cdot \dfrac{d}{dx}(\sin(x^5))$

$+ \sin^2(x^5)(-\sin x^3)\dfrac{d}{dx}(x^3)$

$= \cos x^3 \cdot 2\sin(x^5) \cdot \cos x^5 \cdot 5x^4 - \sin^2(x^5)\sin x^3 \cdot 3x^2$

$= 5x^4 \cdot \cos x^3 \cdot 2\sin(x^5) \cdot \cos x^5$

$- 3x^2 \cdot \sin^2(x^5) \cdot \sin x^3$

$= 5x^4 \cdot \cos x^3 \sin(2x^5) - 3x^2 \cdot \sin^2(x^5)\sin x^3$

4.

રીત 1 :

$I = \displaystyle\int \dfrac{\cos x}{1+\cos x}\,dx$

$= \displaystyle\int \dfrac{\cos x}{1+\cos x} \times \dfrac{(1-\cos x)}{(1-\cos x)}\,dx$

$I = \displaystyle\int \dfrac{\cos x - \cos^2 x}{\sin^2 x}\,dx$

$$= \int \frac{\cos x}{\sin^2 x}\, dx - \int \frac{\cos^2 x}{\sin^2 x}\, dx$$

$$= \int \cot x \cdot \operatorname{cosec} x\, dx - \int \cot^2 x\, dx$$

$$= \int \cot x \cdot \operatorname{cosec} x\, dx - \int (\operatorname{cosec}^2 x - 1) dx$$

$$= \int \cot x \cdot \operatorname{cosec} x\, dx - \int \operatorname{cosec}^2 x\, dx + \int 1\, dx$$

$$I = -\operatorname{cosec} x + \cot x + x + c$$

$$I = x - (\operatorname{cosec} x - \cot x) + c$$

$$I = x - \left(\frac{1 - \cos x}{\sin x}\right) + c$$

$$= x - \left(\frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}}\right) + c$$

$$= x - \tan \frac{x}{2} + c$$

રીત 2 :

$$\int \frac{\cos x}{1 + \cos x}\, dx$$

$$= \int \frac{1 + \cos x - 1}{1 + \cos x}\, dx$$

$$= \int \frac{1 + \cos x}{1 + \cos x}\, dx - \int \frac{1}{1 + \cos x}\, dx$$

$$= \int 1\, dx - \int \frac{1}{2 \cos^2 \frac{x}{2}}\, dx$$

$$= \int 1\, dx - \frac{1}{2} \int \sec^2 \frac{x}{2}\, dx$$

$$= x - \frac{1}{2} \frac{\tan \frac{x}{2}}{\frac{1}{2}} + c$$

$$= x - \tan \frac{x}{2} + c$$

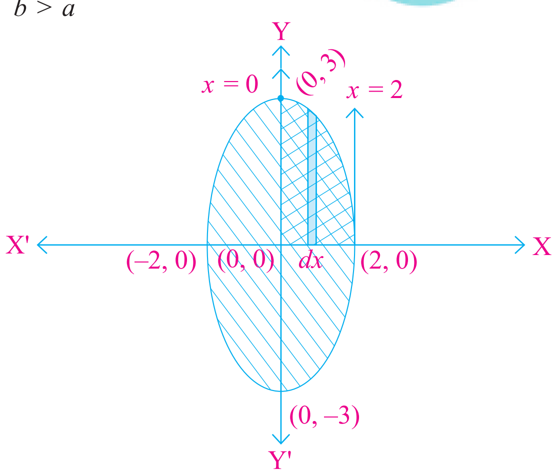
5.

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$a^2 = 4,\ a = 2$

$b^2 = 9,\ b = 3$

$b > a$



આવૃત્ત પ્રદેશનું ક્ષેત્રફળ :

A = 4 × પ્રથમ પ્રદેશ વડે આવૃત્ત ક્ષેત્રફળ

$\therefore$ A = 4|I|

હવે, $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$$\therefore y^2 = 9\left[1 - \frac{x^2}{4}\right] = \frac{9}{4}(4 - x^2)$$

$$\therefore y = \frac{3}{2}\sqrt{4 - x^2}$$

$$I = \int_0^2 y\, dx$$

$$I = \int_0^2 \frac{3}{2}\sqrt{4 - x^2}\, dx$$

$$I = \frac{3}{2}\int_0^2 \sqrt{4 - x^2}\, dx$$

$$= \frac{3}{2}\int_0^2 \sqrt{2^2 - x^2}\, dx$$

$$I = \frac{3}{2}\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$I = \frac{3}{2}\left[\left(\frac{2}{2}(0) + 2\sin^{-1}(1)\right) - (0)\right]$$

$$I = \frac{3}{2} \cdot 2 \cdot \frac{\pi}{2}$$

$$I = \frac{3\pi}{2}$$

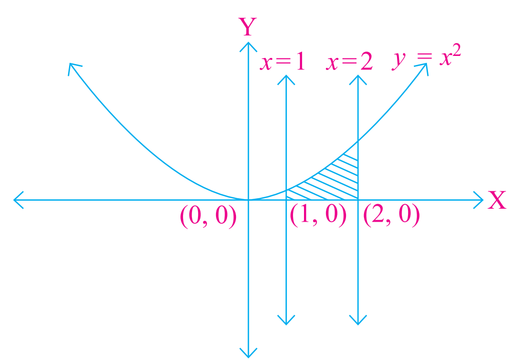
હવે, A = 4|I|

$$= 4\left|\frac{3\pi}{2}\right|$$

$\therefore$ A = 6π ચોરસ એકમ

6.

$x^2 = y$



આવૃત્ત પ્રદેશનું ક્ષેત્રફળ,

A = |I|

$$\therefore I = \int_1^2 y\, dx$$

$$\therefore I = \int_1^2 x^2\, dx$$

$$\therefore I = \left[\frac{x^3}{3}\right]_1^2$$

$\therefore \quad I = \frac{1}{3}\left((2)^3 - (1)^3\right)$

$\therefore \quad I = \frac{7}{3}$

હવે, $A = |I| = \left|\frac{7}{3}\right|$

$\therefore \quad A = \frac{7}{3}$ ચોરસ એકમ

7.

$y \log y \, dx - x \, dy = 0$

$\therefore x \, dy = y \log y \, dx$

$\therefore \quad \frac{dy}{y \log y} = \frac{dx}{x}$

→ બંને બાજુ સંકલન કરતાં,

$\therefore \quad \int \frac{dy}{y \log y} = \int \frac{dx}{x}$

$\therefore \quad \int \frac{\frac{1}{y} dy}{\log y} = \int \frac{dx}{x}$

$\therefore \quad \int \frac{\frac{d}{dy}(\log y)}{\log y} dy = \int \frac{dx}{x}$

$\therefore \quad \log |\log (y)| = \log |x| + \log |c|$

$\therefore \quad \log |\log y| = \log |x \cdot c|$

$\therefore \quad \log y = x \cdot c$

$\therefore \quad y = e^{xc};$

જે આપેલ વિકલ સમીકરણનો વ્યાપક ઉકેલ છે.

8.

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c}$

$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$

$= 3\hat{i} - 3\hat{j} + 2\hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c}$ ને સમાંતર સદિશ,

$= \frac{2\vec{a} - \vec{b} + 2\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|}$

$= \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{9+9+4}}$

$= \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$

9.

અહીં; $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$

$\therefore \quad L : \vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(2\hat{i} + 5\hat{j} - 3\hat{k})$

$\vec{b_1} = 2\hat{i} + 5\hat{j} - 3\hat{k}$

તથા $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

$\therefore \quad M : \vec{r} = (-2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 8\hat{j} + 4\hat{k})$

$\vec{b_2} = -\hat{i} + 8\hat{j} + 4\hat{k}$

$\vec{b_1} \cdot \vec{b_2} = (2\hat{i} + 5\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 8\hat{j} + 4\hat{k})$

$= -2 + 40 - 12$

$= 26$

$|\vec{b_1}| = \sqrt{4+9+25}$

$= \sqrt{38}$

$|\vec{b_2}| = \sqrt{1+64+16}$

$= \sqrt{81}$

$= 9$

જો L અને M વચ્ચેનો ખૂણો α હોય તો,

$\cos \alpha = \frac{|\vec{b_1} \cdot \vec{b_2}|}{|\vec{b_1}||\vec{b_2}|}$

$\cos \alpha = \frac{|26|}{\sqrt{38}\, 9}$

$\therefore \quad \alpha = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$

આથી, બે રેખાઓ વચ્ચેના ખૂણાનું માપ $\cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$ મળે.

10.

$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$

$\therefore L : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 2k\hat{j} + 2\hat{k}), \lambda \in R$

$\vec{b_1} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$

વળી, $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$

$\therefore M : \vec{r} = (\hat{i} + \hat{j} + 6\hat{k}) + \mu(3k\hat{i} + \hat{j} - 5\hat{k}), \mu \in R$

$\vec{b_2} = 3k\hat{i} + \hat{j} - 5\hat{k}$

બંને રેખાઓ પરસ્પર લંબ છે.

$\therefore \quad \vec{b_1} \cdot \vec{b_2} = 0$

$\therefore \quad (-3\hat{i} + 2k\hat{j} + 2\hat{k}) \cdot (3k\hat{i} + \hat{j} - 5\hat{k}) = 0$

$\therefore \quad -9k + 2k - 10 = 0$

$\therefore \quad -7k = 10$

$\therefore \quad k = \frac{-10}{7}$

11.

કુટુંબના ફોટા માટે માતા-પિતા અને પુત્ર યાદૃચ્છિક રીતે એકસાથે હારમાં ઊભા રહે છે.

ધારો કે માતા $\rightarrow$ M

પિતા $\rightarrow$ F

પુત્ર $\rightarrow$ S

$\therefore$ ત્રણ વ્યક્તિઓને હારમાં ગોઠવવાના પ્રકાર $= {}_3P_3$

$\therefore$ શક્ય પરિણામો $= {}_3P_3 = 3! = 6$

$\therefore$ S = {(M, F, S), (M, S, F), (F, M, S), (F, S, M), (S, M, F), (S, F, M)}

ઘટના E : પુત્ર એક છેડા પર છે.

E = {(M, F, S), (F, M, S), (S, M, F), (S, F, M)}

$\therefore r = 4$

$\therefore P(E) = \frac{r}{n} = \frac{4}{6} = \frac{2}{3}$

ઘટના F : પિતા મધ્યમાં છે.

F : {(M, F, S), (S, F, M)}

$\therefore r = 2$

$\therefore P(F) = \frac{2}{6} = \frac{1}{3}$

$E \cap F$ = {(M, F, S), (S, F, M)}

$\therefore r = 2$

$\therefore P(E \cap F) = \frac{1}{3}$

$\therefore P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$

12.

બે પાસા ફેંકતા $n = 36$

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

ઘટના A : બંને પાસા પર ભિન્ન સંખ્યા મળે.

$\therefore r = 30$

$\therefore P(A) = \frac{30}{36} = \frac{5}{6}$

ઘટના B : પાસા પરની સંખ્યાનો સરવાળો 4 છે.

B = {(1, 3), (2, 2), (3, 1)}

$A \cap B$ = {(1, 3), (3, 1)}

$\therefore r = 2$

$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

$\therefore P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{18}}{\frac{5}{6}} = \frac{1}{15}$

વિભાગ-B

13.

અહીં $A = R - \{3\}$, $B = R - \{1\}$, $f(x) = \left(\frac{x-2}{x-3}\right)$

$\forall\, x_1, x_2 \in A$, $f(x_1) = f(x_2)$

$\therefore \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$\therefore (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\therefore x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$

$\therefore x_1 = x_2$

$\therefore f$ એ એક-એક વિધેય છે.

ધારો કે, $y \in B = R - \{1\}$

$y = f(x)$

$\therefore y = \frac{x-2}{x-3}$

$\therefore y(x - 3) = x - 2$

$\therefore yx - 3y = x - 2$

$\therefore yx - x = 3y - 2$

$\therefore x(y - 1) = 3y - 2$

$\therefore x = \frac{3y-2}{y-1} \in R - \{3\}$ (પ્રદેશ)

હવે, $f(x) = f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3}$

$= \frac{3y - 2 - 2y + 2}{3y - 2 - 3y + 3} = y$

$\therefore \forall\, y \in B = R - \{1\}$ માટે

$x = \frac{3y-2}{y-1} \in A = R - \{3\}$ એવો મળે છે,

કે જેથી $f(x) = y$ થાય છે.

$\therefore f$ એ વ્યાપ્ત વિધેય છે.

નોંધ : $f : R - \left\{-\frac{d}{c}\right\} \rightarrow R - \left\{\frac{a}{c}\right\}$;

$f(x) = \frac{ax+b}{cx+d}$ એ હંમેશાં એક-એક છે અને વ્યાપ્ત વિધેય છે.

14.

ટ્રસ્ટ પાસે કુલ ભંડોળ ₹ 30,000 છે.

ધારો કે ટ્રસ્ટ પ્રથમ બોન્ડ ₹ x નું રોકાણ કરે છે.

∴ ટ્રસ્ટે બીજા બોન્ડમાં કરેલું રોકાણ $(30000 - x)$ ₹

પ્રથમ બોન્ડ પ્રતિવર્ષ 5% વ્યાજ આપે છે અને

બીજો બોન્ડ પ્રતિવર્ષ 7% વ્યાજ આપે છે.

(a) વાર્ષિક ₹ 1,800 વ્યાજ મેળવવું છે.

$5\% = ₹\ \frac{5}{100}$

$7\% = ₹\ \frac{7}{100}$

$$\therefore [x \quad 30000 - x]\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = [1800]$$

$$\therefore \left[\frac{5}{100}x + \frac{7}{100}(30000 - x)\right] = [1800]$$

$$\therefore \left[\frac{5x + 210000 - 7x}{100}\right] = [1800]$$

$- 2x + 210000 = 180000$

$\therefore 2x = 30000$

$\therefore x = 15000$

આમ, વાર્ષિક વ્યાજ ₹ 1,800 મેળવવા માટે પ્રથમ બોન્ડમાં ₹ 15,000 અને બીજા બોન્ડમાં

30000 – 15000 = ₹ 15,000 રોકવા પડે.

(b) વાર્ષિક ₹ 2,000 વ્યાજ મેળવવું છે.

$$\therefore [x \quad 30000 - x]\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = [2000]$$

$$\therefore \left[\frac{5x}{100} + \frac{7}{100}(30000 - x)\right] = [2000]$$

$$\therefore \left[\frac{5x}{100} + \frac{7}{100}(30000 - x)\right] = [2000]$$

$\therefore 5x + 210000 - 7x = 200000$

$210000 - 200000 = 2x$

$\therefore 2x = 10000$

$\therefore x = ₹\ 5{,}000$

આમ, વાર્ષિક વ્યાજ ₹ 2,000 મેળવવા માટે પ્રથમ બોન્ડમાં ₹ 5,000 અને

બીજા બોન્ડમાં = 30000 – 5000

= ₹ 25,000નું રોકાણ કરવું પડે.

15.

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

yz નો સહઅવયવ $A_{13} = (-1)^4 \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix}$

$= (1)(z - y)$

$= (z - y)$

zx નો સહઅવયવ $A_{23} = (-1)^5 \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix}$

$= (-1)(z - x)$

$= x - z$

xy નો સહઅવયવ $A_{33} = (-1)^6 \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix}$

$= (1)(y - x)$

$= y - x$

$\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

$= (yz)(z - y) + (zx)(x - z) + (xy)(y - x)$

$= yz^2 - y^2z + zx^2 - z^2x + xy^2 - x^2y$

$= z(x^2 - y^2) + z^2(y - x) + xy(y - x)$

$= z[(x - y)(x + y)] + z^2(y - x) + xy(y - x)$

$= (y - x)(- z(x + y) + z^2 + xy)$

$= (y - x)(-zx - yz + z^2 + xy)$

$= (y - x)(z(z - x) - y(z - x))$

$= (y - x)(z - x)(z - y)$

$= (x - y)(y - z)(z - x)$

16.

ધારો કે, $u = (logx)^x$ અને $v = x^{logx}$

$\therefore y = u + v$

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \text{......... (1)}$$

અહીં, $u = (logx)^x$ ની

બંને બાજુ log લેતાં,

$log\ u = x\ log(logx)$

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$\therefore \frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}log(logx) + log(logx)\frac{d}{dx}x$$

$$= x \times \frac{1}{xlogx} + log(logx)$$

$$\therefore \frac{du}{dx} = u\left[\frac{1}{log x} + log(logx)\right]$$

$$\frac{du}{dx} = (logx)^x\left[\frac{1}{log x} + log(logx)\right] \qquad \text{...... (2)}$$

હવે, $v = x^{logx}$ ની

બંને બાજુ log લેતાં,

$log\ v = log\ x \cdot log\ x$

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$\frac{1}{v}\frac{dv}{dx} = logx\frac{d}{dx}logx + logx \cdot \frac{d}{dx}log\ x$$

$$= logx \times \frac{1}{x} + log\ x \times \frac{1}{x}$$

$\therefore \quad \frac{1}{v}\frac{dv}{dx} = \frac{2logx}{x}$

$\therefore \quad \frac{dv}{dx} = v\left[\frac{2logx}{x}\right]$

$\therefore \quad \frac{dv}{dx} = x^{logx}\left[\frac{2logx}{x}\right]$ ...... (3)

પરિણામ (2) અને (3) ની કિંમત પરિણામ 1 માં મૂકતાં,

$$\therefore \quad \frac{dy}{dx} = (logx)^x\left[\frac{1}{log\,x} + log(log\,x)\right] + x^{logx}\left[\frac{2logx}{x}\right]$$

$$= (logx)^{x-1} + (logx)^x \cdot log(logx) + x^{logx-1}\,[2log\;x]$$

$$\frac{dy}{dx} = (logx)^{x-1}[1 + logx \cdot log(logx)] + 2x^{log\,x-1}(logx)$$

17.

$$y = \frac{4\,sin\,\theta}{(2 + cos\,\theta)} - \theta,\ \theta \in \left[0, \frac{\pi}{2}\right]$$

$$\therefore \quad \frac{dy}{d\theta} = \frac{(2 + cos\,\theta)(4\,cos\,\theta) - 4\,sin\,\theta\,(-sin\,\theta)}{(2 + cos\,\theta)^2} - 1$$

$$= \frac{8\,cos\,\theta + 4\,cos^2\theta + 4\,sin^2\theta}{(2 + cos\,\theta)^2} - 1$$

$$= \frac{8\,cos\,\theta + 4(cos^2\theta + sin^2\theta) - (2 + cos\,\theta)^2}{(2 + cos\,\theta)^2}$$

$$= \frac{8\,cos\,\theta + 4 - 4 - 4\,cos\,\theta - cos^2\theta}{(2 + cos\,\theta)^2}$$

$$= \frac{4\,cos\,\theta - cos^2\theta}{(2 + cos\,\theta)^2}$$

$$\frac{dy}{d\theta} = \frac{cos\,\theta\,(4 - cos\,\theta)}{(2 + cos\,\theta)^2}$$

અહીં, $\theta \in \left[0, \frac{\pi}{2}\right] \Rightarrow cos\;\theta \geq 0$

$\Rightarrow (4 - cos\;\theta) > 0$

$\Rightarrow (2 + cos\;\theta)^2 > 0$

$\Rightarrow \frac{cos\,\theta\,(4 - cos\,\theta)}{(2 + cos\,\theta)^2} \geq 0$

$\Rightarrow \frac{dy}{d\theta} \geq 0$

$\therefore \left[0, \frac{\pi}{2}\right]$ પર f વધતું વિધેય છે.

18.

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

સમાંતરબાજુ ચતુષ્કોણનું ક્ષેત્રફળ

$\Delta = |\vec{a} \times \vec{b}|$ ... (1)

હવે, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$

$\vec{a} \times \vec{b} = 20\hat{i} + 5\hat{j} - 5\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{400 + 25 + 25}$

$= \sqrt{450}$

$= 15\sqrt{2}$

પરિણામ (1) પરથી, $\Delta = 15\sqrt{2}$ ચો. એકમ

19.

બે રેખાઓ સમાંતર છે.

આપણી પાસે $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$,

$\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$ અને

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ છે.

આથી, રેખાઓ વચ્ચેનું અંતર

$$d = \left|\frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|}\right|$$

$$= \left|\frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}}{\sqrt{4 + 9 + 36}}\right|$$

$$= \frac{|-9\hat{i} + 14\hat{j} - 4\hat{k}|}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{7} \text{ એકમ}$$

20.

$x + 2y \leq 120$ ...(1)

$x + y \geq 60$ ...(2)

$x - 2y \geq 0$ ...(3)

$x \geq 0$

$y \geq 0$

હેતુલક્ષી વિધેય $Z = 5x + 10y$

$x + 2y = 120$ ... (i)

x	0	120	(0, 60) ×
y	60	0	(120, 0) ✓

$x + y = 60$ ... (ii)

x	0	60	(0, 60) ×
y	60	0	(60, 0) ✓

$x - 2y = 0$ ... (iii)

x	0	2	(0, 0) ×
y	0	0	(2, 1)

(i) અને (ii)નો ઉકેલ,

$$\begin{array}{rl} x + 2y &= 120 \\ x + y &= 60 \\ - \quad - & \quad - \\ \hline y &= 60 \end{array} \quad (0, 60) \times$$

$\therefore x = 0$

(ii) અને (iii)નો ઉકેલ,

$$\begin{array}{rl} x + y &= 60 \\ x - 2y &= 0 \\ - \quad + & \quad - \\ \hline 3y &= 60 \end{array} \quad (40, 20) ✓$$

$\therefore y = 20$

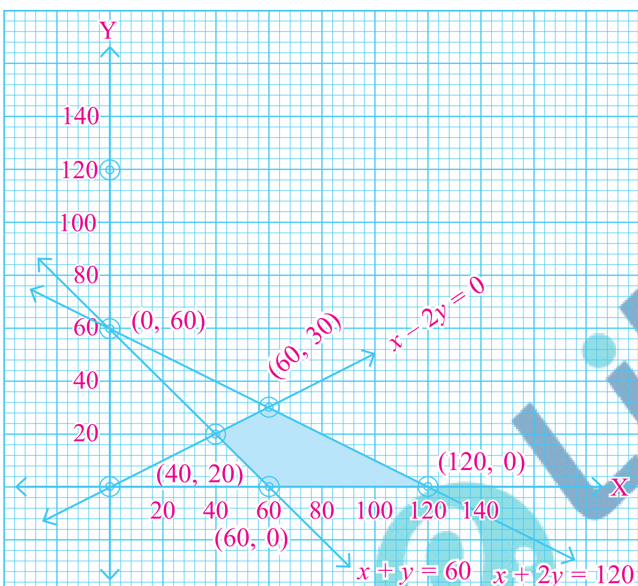
$\therefore x = 40$

(i) અને (iii)નો ઉકેલ,

$\therefore 4y = 120$

$\therefore y = 30$ $\qquad$ $\therefore x = 60$

(60, 30) ✓ $\qquad$ (0, 0) ×



આકૃતિમાં આપેલ અસમતાઓનો આલેખ દર્શાવ્યો છે જે સિમિત છે. શક્ય ઉકેલપ્રદેશનાં શિરોબિંદુઓ (60, 0), (120, 0), (60, 30) અને (40, 20) મળે.

શક્ય ઉકેલ પ્રદેશના શિરોબિંદુ	$Z = 5x + 10y$
(60, 30)	600 ← મહત્તમ
(40, 20)	400
(60, 0)	300 ← ન્યૂનતમ
(120, 0)	600 ← મહત્તમ

બિંદુઓ (120, 0) અને (60, 30) આગળ Zનું મહત્તમ મૂલ્ય 600 મળે તથા ન્યૂનતમ મૂલ્ય 300 બિંદુ (60, 0) આગળ મળે.

21.

➪ છાત્રાલયમાં રહેતા વિદ્યાર્થીઓ 60% છે.

ઘટના A : વિદ્યાર્થી છાત્રાલયમાં રહેતો હોય

$$P(A) = \frac{60}{100}$$

ઘટના B : વિદ્યાર્થી છાત્રાલયમાં રહેતો ન હોય

$$P(B) = \frac{40}{100}$$

ઘટના E : વિદ્યાર્થી A_1 ગ્રેડ મેળવે.

$$\begin{aligned} P(E) &= P(A) \cdot P(E \mid A) + P(B) \cdot P(E \mid B) \\ &= \frac{60}{100} \times \frac{30}{100} + \frac{40}{100} \times \frac{20}{100} \\ &= \frac{18}{100} + \frac{8}{100} \\ &= \frac{26}{100} \end{aligned}$$

$$\begin{aligned} P(A \mid E) &= \frac{P(A) \cdot P(E|A)}{P(E)} \\ &= \frac{\frac{60}{100} \times \frac{30}{100}}{\frac{26}{100}} \\ &= \frac{9}{13} \end{aligned}$$

વિભાગ-C

22.

➪ અહીં, $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$$A^T = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\begin{aligned} P &= \frac{1}{2}(A + A^T) \\ &= \frac{1}{2}\left\{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right\} \\ &= \frac{1}{2}\begin{bmatrix} 3+3 & 3-2 & -1-4 \\ -2+3 & -2-2 & 1-5 \\ -4-1 & -5+1 & 2+2 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} \end{aligned}$$

$\therefore$ P $= P^T$

$\therefore$ P એ સંમિત શ્રેણિક છે.

$$Q = \frac{1}{2}(A - A^T)$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 3-3 & 3+2 & -1+4 \\ -2-3 & -2+2 & 1+5 \\ -4+1 & -5-1 & 2-2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

$$Q^T = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

$\therefore$ Q $= -Q^T$

$\therefore$ Q એ વિસંમિત શ્રેણિક છે.

$$P + Q = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$= A$

23.

➪ $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

$= 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2)$

$= 0 + 3(-2) + 5(1)$

$= -6 + 5$

$= -1 \neq 0$

$\therefore$ A^{-1} નું અસ્તિત્વ છે.

adj A મેળવવા માટે,

2 નો સહઅવયવ $A_{11} = (-1)^2 \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix}$

$= 1(-4 + 4)$

$= 0$

–3 નો સહઅવયવ $A_{12} = (-1)^3 \begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix}$

$= (-1)(-6 + 4)$

$= 2$

5 નો સહઅવયવ $A_{13} = (-1)^4 \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$

$= 1(3 - 2)$

$= 1$

3 નો સહઅવયવ $A_{21} = (-1)^3 \begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix}$

$= (-1)(6 - 5)$

$= -1$

2 નો સહઅવયવ $A_{22} = (-1)^4 \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix}$

$= 1(-4 - 5)$

$= -9$

–4 નો સહઅવયવ $A_{23} = (-1)^5 \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}$

$= (-1)(2 + 3)$

$= -5$

1 નો સહઅવયવ $A_{31} = (-1)^4 \begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix}$

$= 1(12 - 10)$

$= 2$

1 નો સહઅવયવ $A_{32} = (-1)^5 \begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix}$

$= (-1)(-8 - 15)$

$= 23$

–2 નો સહઅવયવ $A_{33} = (-1)^6 \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix}$

$= 1(4 + 9)$

$= 13$

$$\therefore \; adj\, A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\, adj\, A$$

$$= \frac{1}{(-1)}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

હવે, $2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

શ્રેણિક સ્વરૂપે લખતાં,

$$\therefore \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$\therefore$ AX = B

જ્યાં, $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$\therefore$ $X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

ઉકેલ : $x = 1, y = 2, z = 3$

24.

$(x - a)^2 + (y - b)^2 = c^2$ .... (1)

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$2(x - a) + 2(y - b)\frac{dy}{dx} = 0$$

$$\therefore (x - a) + (y - b)\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = \frac{-(x-a)}{y-b} \quad .... (2)$$

હવે, બંને બાજુ x પ્રત્યે પુનઃ વિકલન કરતાં,

$$\frac{d^2y}{dx^2} = -\left[\frac{(y-b)(1)-(x-a)\cdot\frac{dy}{dx}}{(y-b)^2}\right]$$

$$= -\left[\frac{(y-b)+\frac{(x-a)(x-a)}{(y-b)}}{(y-b)^2}\right] \quad (\because \text{ પરિણામ-2})$$

$$= -\left[\frac{(y-b)^2+(x-a)^2}{(y-b)^3}\right]$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-c^2}{(y-b)^3} \quad (\because \text{ પરિણામ-1})$$

હવે, $$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1+\left(\frac{x-a}{y-b}\right)^2\right]^{\frac{3}{2}}}{\frac{-c^2}{(y-b)^3}}$$

$$= \frac{\left[\frac{(y-b)^2+(x-a)^2}{(y-b)^2}\right]^{\frac{3}{2}}}{\frac{-c^2}{(y-b)^3}}$$

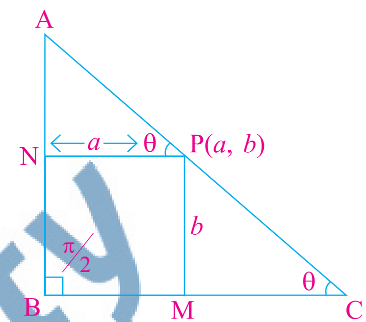
$$= \frac{-[c^2]^{\frac{3}{2}}}{[(y-b)^2]^{\frac{3}{2}}} \times \frac{(y-b)^3}{[c^2]}$$

$$= \frac{-c^3}{(y-b)^3} \times \frac{(y-b)^3}{c^2}$$

$$= -c, \quad c > 0$$

જે a અને b પર આધારિત ન હોય તેવો અચળ છે.

25.



કાટકોણ ΔABCમાં $\overline{AC}$ કર્ણ છે.

કર્ણ પરનું બિંદુ P(a, b) છે.

અહીં, $\overline{PM} \perp \overline{BC}$ તથા $\overline{PN} \perp \overline{AB}$

ધારો કે, ∠APN = ∠PCM = θ

ΔABC કાટકોણ ત્રિકોણ હોવાથી, $\theta \in \left(0, \frac{\pi}{2}\right)$

→ કાટકોણ ΔAPNમાં,

$$\cos\theta = \frac{PN}{AP} = \frac{a}{AP}$$

$$\therefore \quad AP = \frac{a}{\cos\theta}$$

$$\therefore \quad AP = a\sec\theta$$

→ કાટકોણ ΔPMCમાં,

$$\sin\theta = \frac{PM}{PC} = \frac{b}{PC}$$

$$\therefore \quad PC = \frac{b}{\sin\theta}$$

$$\therefore \quad PC = b\,\mathrm{cosec}\,\theta$$

હવે, AC = AP + PC

$\therefore$ AC $= a\sec\theta + b\,\mathrm{cosec}\,\theta$ .... (1)

$f(\theta) = a\sec\theta + b\,\mathrm{cosec}\,\theta$

$\therefore \quad f'(\theta) = a \sec\theta \cdot \tan\theta - b \operatorname{cosec}\theta \cdot \cot\theta$

$\therefore \quad f''(\theta) = a(\sec\theta \cdot \sec^2\theta + \tan\theta \cdot \sec\theta \cdot \tan\theta)$
$\qquad - b(\operatorname{cosec}\theta\,(-\operatorname{cosec}^2\theta)$
$\qquad + \cot\theta(-\operatorname{cosec}\theta \cot\theta))$

$\therefore \quad f''(\theta) = a(\sec^3\theta + \sec\theta \cdot \tan^2\theta)$
$\qquad + b(\operatorname{cosec}^3\theta + \operatorname{cosec}\theta \cot^2\theta)$

$\therefore \quad f''(\theta) > 0 \qquad \left(\because 0 < \theta < \frac{\pi}{2}\right)$

→ કર્ણની લંબાઇ ન્યૂનતમ મેળવવા માટે,

$f'(\theta) = 0$

$\therefore \quad a \sec\theta \tan\theta - b \operatorname{cosec}\theta \cot\theta = 0$

$\therefore \quad a \sec\theta \tan\theta = b \operatorname{cosec}\theta \cot\theta$

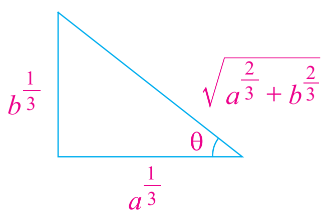
$\therefore \quad \frac{a}{\cos\theta} \frac{\sin\theta}{\cos\theta} = \frac{b}{\sin\theta} \frac{\cos\theta}{\sin\theta}$

$\therefore \quad \frac{a}{\cos^3\theta} = \frac{b}{\sin^3\theta}$

$\therefore \quad \frac{\sin^3\theta}{\cos^3\theta} = \frac{b}{a}$

$\therefore \quad \tan^3\theta = \frac{b}{a}$

$\therefore \quad \tan\theta = \left(\frac{b}{a}\right)^{\frac{1}{3}}$



$\sec\theta = \frac{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}{a^{\frac{1}{3}}}$

$\operatorname{cosec}\theta = \frac{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}{b^{\frac{1}{3}}}$

આ કિંમતો પરિણામ (1) માં મૂકતાં,

$\therefore \text{AC} = \frac{a\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{1}{2}}}{a^{\frac{1}{3}}} + \frac{b\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{1}{2}}}{b^{\frac{1}{3}}}$

$\therefore \text{AC} = \left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{1}{2}} \left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{1}$

$\therefore \text{AC} = \left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$

$\therefore$ કર્ણની ન્યૂનતમ લંબાઇ $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$ છે.

26.

➪ $\text{I} = \int \frac{\sqrt{x^2+1}\,[\log(x^2+1) - 2\log x]}{x^4}\,dx$

$= \int \frac{\sqrt{x^2+1}\,[\log(x^2+1) - \log x^2]}{x^4}\,dx$

$= \int \frac{\sqrt{x^2+1}}{x^4} \log\left(\frac{x^2+1}{x^2}\right) dx$

$= \int \frac{x\sqrt{1+\frac{1}{x^2}}}{x^4} \log\left(1+\frac{1}{x^2}\right) dx$

$\text{I} = \int \frac{\sqrt{1+\frac{1}{x^2}}}{x^3} \log\left(1+\frac{1}{x^2}\right) dx$

હવે, $1 + \frac{1}{x^2} = t^2$ આદેશ લેતાં,

$\therefore \frac{-2}{x^3}\,dx = 2t \cdot dt$

$\therefore \frac{dx}{x^3} = -t\,dt$

$\text{I} = \int t \cdot \log(t^2)\,(-t\,dt)$

$= \int -2t^2 \cdot \log t\,dt$

$\text{I} = -2 \int t^2 \cdot \log t\,dt$

$\text{I} = -2\,\text{I}_1$ ... (1)

હવે, $\text{I}_1 = \int t^2 \cdot \log t\,dt$

$u = \log t \quad ; \quad v = t^2$

$\therefore \frac{du}{dt} = \frac{1}{t}$

$\text{I}_1 = \log t \int t^2\,dt - \int \left[\frac{1}{t} \int t^2\,dt\right] dt$

$= \frac{\log t \cdot t^3}{3} - \int \frac{1}{t} \cdot \frac{t^3}{3}\,dt$

$= \frac{\log t \cdot t^3}{3} - \frac{1}{3} \int t^2\,dt$

$\text{I}_1 = \frac{\log t \cdot t^3}{3} - \frac{t^3}{9} + c$

હવે, $1 + \frac{1}{x^2} = t^2$ હોવાથી,

$\therefore t = \sqrt{1+\frac{1}{x^2}}$ અને $t^3 = \left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}$

$\text{I}_1 = \frac{t^3}{3}\left[\log t - \frac{1}{3}\right] + c$

$\text{I}_1 = \frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}} \left[\log\left(1+\frac{1}{x^2}\right)^{\frac{1}{2}} - \frac{1}{3}\right] + c_1$

$\text{I} = \frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}} \left[\frac{1}{2}\log\left[1+\frac{1}{x^2}\right] - \frac{1}{3}\right] + c_1$

I_1 ની કિંમત પરિણામ (1) માં મૂકતાં,

$\text{I} = \frac{-2}{3}\left[1+\frac{1}{x^2}\right]^{\frac{3}{2}} \left[\frac{1}{2}\log\left(1+\frac{1}{x^2}\right) - \frac{1}{3}\right] + c$

$\text{I} = \frac{-1}{3}\left[1+\frac{1}{x^2}\right]^{\frac{3}{2}} \left[\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + c$

27.

રીત 1 :

આપેલ વિકલ સમીકરણ નીચે પ્રમાણે લખી શકાય :

$$\left[xy\sin\left(\frac{y}{x}\right)-x^2\cos\left(\frac{y}{x}\right)\right]dy = \left[xy\cos\left(\frac{y}{x}\right)+y^2\sin\left(\frac{y}{x}\right)\right]dx$$

$$xy\cdot\sin\left(\frac{y}{x}\right)dy - y^2\sin\left(\frac{y}{x}\right)dx = xy\cdot\cos\left(\frac{y}{x}\right)dx + x^2\cos\left(\frac{y}{x}\right)dy$$

$$\therefore \frac{dy}{dx} = \frac{xy\cos\left(\frac{y}{x}\right)+y^2\sin\left(\frac{y}{x}\right)}{xy\sin\left(\frac{y}{x}\right)-x^2\cos\left(\frac{y}{x}\right)}$$

જમણી બાજુ અંશ અને છેદને x^2 વડે ભાગતાં,

$$\therefore \frac{dy}{dx} = \frac{\frac{y}{x}\cos\left(\frac{y}{x}\right)+\left(\frac{y^2}{x^2}\right)\sin\left(\frac{y}{x}\right)}{\frac{y}{x}\sin\left(\frac{y}{x}\right)-\cos\left(\frac{y}{x}\right)} \quad \text{... (1)}$$

સમીકરણ (1) એ $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$ પ્રકારનું સમપરિમાણ વિકલ સમીકરણ છે.

→ આ સમીકરણનો ઉકેલ મેળવવા માટે આપણે $y = vx$ લઈશું.

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{... (2)}$$

$$\therefore v + x\frac{dv}{dx} = \frac{v\cos v + v^2\sin v}{v\sin v - \cos v}$$

((1) અને (2)ના ઉપયોગથી)

$$\therefore x\frac{dv}{dx} = \frac{2v\cos v}{v\sin v - \cos v}$$

$$\therefore \left(\frac{v\sin v - \cos v}{v\cos v}\right)dv = \frac{2dx}{x}$$

માટે, $\int\left(\frac{v\sin v - \cos v}{v\cos v}\right)dv = 2\int\frac{1}{x}dx$

$$\therefore \int \tan v\, dv - \int\frac{1}{v}dv = 2\int\frac{1}{x}dx$$

$$\therefore \log|\sec v| - \log|v| = 2\log|x| + \log|c_1|$$

$$\therefore \log\left|\frac{\sec v}{vx^2}\right| = \log|c_1|$$

$$\therefore \frac{\sec v}{vx^2} = \pm c_1$$

→ સમીકરણ (3) માં $v = \frac{y}{x}$ મૂકતાં,

$$\therefore \frac{\sec\left(\frac{y}{x}\right)}{\left(\frac{y}{x}\right)(x^2)} = c \qquad \text{જ્યાં, } c = \pm c_1$$

$$\therefore \sec\left(\frac{y}{x}\right) = cxy$$

આ આપેલ વિકલ સમીકરણનો વ્યાપક ઉકેલ છે.

રીત 2 :

$$\left(\frac{x\,dy - y\,dx}{x^2}\right)y\sin\frac{y}{x} = \left(\frac{y\,dx + x\,dy}{x}\right)\cos\frac{y}{x}$$

$$\therefore d\left(\frac{y}{x}\right)\sin\frac{y}{x} = \frac{d(xy)}{xy}\cos\frac{y}{x}$$

$$\therefore d\left(\frac{y}{x}\right)\tan\frac{y}{x} = \frac{d(xy)}{xy}$$

$$\therefore \log\left|\sec\frac{y}{x}\right| = \log|cxy|$$

$$\therefore \sec\frac{y}{x} = cxy$$